

Closing Wed: HW_7A,7B,7C (7.8, 8.1)
Midterm 2 is Thursday, covers: 6.4, 6.5,
7.1-7.5, 7.7, 7.8, 8.1

A Very Brief Exam 2 Review

Integration!

1. The 4 special methods

By parts: $x e^{3x}$, $x \sin(4x)$, $x^2 \cos(5x)$,
 $\frac{\ln(x-1)}{x^2}$, $x \tan^{-1}(x)$.

Trig: $\sin^3(x) \cos(x)$, $\cos^4(x)$,
 $\sec^3(2x) \tan^3(2x)$

Trig sub: $\frac{1}{x\sqrt{x^2-9}}$, $\frac{1}{(4-x^2)^{3/2}}$,
 $\sqrt{x^2 + 6x + 10}$.

Part Frac: $\frac{x+2}{(x-1)(x-3)}$, $\frac{4x}{(x-1)^2(x-3)}$, $\frac{5}{x(x^2+4)}$,
 $\frac{x^2}{x+7}$, $\frac{x-3}{x^2+8x+20}$

2. Substitution and Simplifying

Try $u = \sqrt{x}$, $u = \text{inside}$, $u = e^x$, $u = \text{trig}$.

Know essential toolbox of trig facts, such
as $\tan(x) = \frac{\sin(x)}{\cos(x)}$, $\sec(x) = \frac{1}{\cos(x)}$.

Square identities, half-angle.

3. Improper Integrals:

- Rewrite as a limit!!
- Integrate
- Take limit

4. Trapezoid/Simpson Rules

- Set up integral, then compute width
and label tickmarks.
- Use formula.

5. New Applications

a) **Average value** = $\frac{1}{b-a} \int_a^b f(x) dx$

b) **Arc Length** = $\int_a^b \sqrt{1 + (f'(x))^2} dx$

c) **Work** = $\int_a^b (\text{Force})(\text{Dist})$

(i) If $f(x)$ = “force formula at x ”,
(Spring, leaky bucket, ...)

then

$$\text{Force} = f(x), \text{Dist} = \Delta x:$$

$$\text{Work} = \int_a^b f(x) dx$$

(ii) *Chain/Cable*: k = force/length

If you label top: $x = 0$, then

$$\text{Force} = k \Delta x, \text{Dist} = x,$$

$$\text{Work} = \int_a^b k x dx$$

(iii) *Pumping*: k = force/volume

If bottom is $y = 0$ and top is $y = b$,

$$\text{Force} = k(\text{Area})\Delta y, \text{Dist} = b - y$$

$$\text{Work} = \int_a^b k(\text{Area})(b - y) dy$$

Random Problems from Old Finals

Integrals:

1. $\int \frac{1-x}{\sqrt{1-x^2}} dx$

2. $\int \frac{x^2 - x + 8}{x^3 + 4x} dx$

3. $\int 2x \ln(x+5) dx$

4. $\int \cos^3(x) dx$

5. $\int_0^2 \frac{1}{\sqrt{x^2 + 2x + 4}} dx$

6. $\int_1^3 \frac{1}{x^2 + x^3} dx$

7. $\int \tan^2 x \sec^4(x) dx$

8. $\int \frac{1}{(1 + \sqrt{x})^3} dx$

9. $\int \sin(x) \sqrt{\cos(x)} dx$

Improper:

1.
$$\int_1^2 \frac{x}{\sqrt{x-1}} dx$$

2.
$$\int_{-3}^{\infty} x e^{-x} dx$$

3.
$$\int_1^{\infty} \frac{1}{\sqrt{x}(1+x)} dx$$

Approximation:

1. Use Simpson's Method with $n = 4$ subdivision to approximate the value of the arc length of $y = x^3$ from $x = 0$ to $x = 4$.

Applications:

1. Find the average value of $\cos^3(x)$ on the interval 0 to $\pi/2$.
2. A spring has natural length of 30 cm from the wall. It requires 2 J of work to stretch it from 40 cm to 45cm (from the wall). How far beyond its natural length will a force of 64 N keep the spring stretched?
3. A 1600 lb elevator is suspended by a 200 ft cable that weighs 10 lb/ft. How much work is required to raise the elevator from the basement to the third floor, a distance of 30 ft?
4. A rope is used to pull a bucket full of water up from a well that is 10 m deep. The rope has a total mass of 5 kg. The bucket has a mass of 11 kg. Find the total work done in lifting the bucket to the top (Recall: Accel. due to gravity is 9.8 m/s^2)

5. A well is in the shape of a cylinder of radius 1 meter and depth 8 meters. It is half full of water. Find the work to pump all the water to the top.

(Recall: Water weighs 9800 N/m^3)

6. The portion of the graph $y = x^2 / 9$ between $x = 0$ and $x = 3$ is rotated about the y -axis to form a container. The container is full of a liquid that has density 100 lbs/ft^3 . Find the work required to pump all the liquid to the top of the container.